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2025 II 22

1100

J-312**(E)****MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)****Time : 3 Hrs.****(8 Pages)****Max. Marks : 80****General instructions :**

*The question paper is divided into **FOUR** sections.*

- (1) **Section A :** *Q. 1 contains **Eight** multiple choice type questions carrying **Two** marks each.
Q. 2 contains **Four** very short answer type questions carrying **One** mark each.*
- (2) **Section B :** *This section contains **Twelve** short answer type questions carrying **Two** marks each.
(Attempt any **Eight**)*
- (3) **Section C :** *This section contains **Twelve** short answer type questions carrying **Three** marks each.
(Attempt any **Eight**)*
- (4) **Section D :** *This section contains **Eight** long answer type questions carrying **Four** marks each.
(Attempt any **Five**)*
- (5) *Use of log table is allowed. Use of calculator is not allowed.*
- (6) *Figures to the right indicate full marks.*

- (7) Use of graph paper is not necessary. Only rough sketch of graph is expected.
- (8) For each multiple choice type of questions, only the first attempt will be considered for evaluation.
- (9) Start answer to each section on a new page.

SECTION – A

Q. 1. Select and write the correct answer of the following multiple choice type of questions : [16]

- (i) If $A = \{1, 2, 3, 4, 5\}$ then which of the following is not true ?
- (a) $\exists x \in A$ such that $x + 3 = 8$
- (b) $\exists x \in A$ such that $x + 2 < 9$
- (c) $\forall x \in A, x + 6 \geq 9$
- (d) $\exists x \in A$ such that $x + 6 < 10$ (2)
- (ii) In ΔABC , $(a + b) \cdot \cos C + (b + c) \cos A + (c + a) \cdot \cos B$ is equal to _____.
- (a) $a - b + c$
- (b) $a + b - c$
- (c) $a + b + c$
- (d) $a - b - c$ (2)
- (iii) If $|\vec{a}| = 5$, $|\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$ then $|\vec{a} \cdot \vec{b}|$ is equal to _____.
- (a) 30 (b) 60
- (c) 40 (d) 45 (2)

- (iv) The vector equation of the line passing through the point having position vector $4\hat{i} - \hat{j} + 2\hat{k}$ and parallel to vector $-2\hat{i} - \hat{j} + \hat{k}$ is given by ____.
- (a) $(4\hat{i} - \hat{j} - 2\hat{k}) + \lambda(-2\hat{i} - \hat{j} + \hat{k})$
 (b) $(4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$
 (c) $(4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-2\hat{i} - \hat{j} - \hat{k})$
 (d) $(4\hat{i} - \hat{j} + 2\hat{k}) + \lambda(-2\hat{i} - \hat{j} + \hat{k})$ (2)

- (v) Let $f(1) = 3$, $f'(1) = -\frac{1}{3}$, $g(1) = -4$ and $g'(1) = -\frac{8}{3}$. The derivative of $\sqrt{[f(x)]^2 + [g(x)]^2}$ w.r.t. x at $x = 1$ is ____.
- (a) $-\frac{29}{25}$ (b) $\frac{7}{3}$
 (c) $\frac{31}{15}$ (d) $\frac{29}{15}$ (2)

- (vi) If the mean and variance of a binomial distribution are 18 and 12 respectively, then n is equal to ____.
- (a) 36 (b) 54
 (c) 18 (d) 27 (2)

- (vii) The value of $\int x^x(1 + \log x) dx$ is equal to ____.
- (a) $\frac{1}{2}(1 + \log x)^2 + c$ (b) $x^{2x} + c$
 (c) $x^x \cdot \log x + c$ (d) $x^x + c$ (2)

- (viii) The area bounded by the line $y = x$, X-axis and the lines $x = -1$ and $x = 4$ is equal to ____.
- (in square units)
- (a) $\frac{2}{17}$ (b) 8
 (c) $\frac{17}{2}$ (d) $\frac{1}{2}$ (2)

Q. 2. Answer the following questions : [4]

(i) Write the negation of the statement : ' $\exists n \in \mathcal{N}$ such that $n+8 > 11$ ' (1)

(ii) Write unit vector in the opposite direction to $\vec{u} = 8\hat{i} + 3\hat{j} - \hat{k}$. (1)

(iii) Write the order of the differential equation

$$\sqrt{1 + \left(\frac{dy}{dx}\right)^2} = \left(\frac{d^2y}{dx^2}\right)^{\frac{3}{2}} \quad (1)$$

(iv) Write the condition for the function $f(x)$, to be strictly increasing, for all $x \in \mathcal{R}$. (1)

SECTION - B

Attempt any EIGHT of the following questions : [16]

Q. 3. Using truth table, prove that the statement patterns $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent. (2)

Q. 4. Find the adjoint of the matrix $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$. (2)

Q. 5. Find the general solution of $\tan^2 \theta = 1$. (2)

Q. 6. Find the co-ordinates of the points of intersection of the lines represented by $x^2 - y^2 - 2x + 1 = 0$. (2)

Q. 7. A line makes angles of measure 45° and 60° with the positive directions of the Y and Z axes respectively. Find the angle made by the line with the positive direction of the X -axis. (2)

- Q. 8. Find the vector equation of the plane passing through the point having position vector $2\hat{i} + 3\hat{j} + 4\hat{k}$ and perpendicular to the vector $2\hat{i} + \hat{j} - 2\hat{k}$. (2)
- Q. 9. Divide the number 20 into two parts such that sum of their squares is minimum. (2)
- Q. 10. Evaluate : $\int x^9 \cdot \sec^2(x^{10}) dx$ (2)
- Q. 11. Evaluate : $\int \frac{1}{25 - 9x^2} dx$ (2)
- Q. 12. Evaluate : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{1}{1 - \sin x} dx$ (2)
- Q. 13. Find the area of the region bounded by the parabola $y^2 = 16x$ and its latus rectum. (2)
- Q. 14. Suppose that X is waiting time in minutes for a bus and its p.d.f. is given by :

$$f(x) = \frac{1}{5}, \text{ for } 0 \leq x \leq 5$$

$$= 0, \text{ otherwise.}$$

Find the probability that :

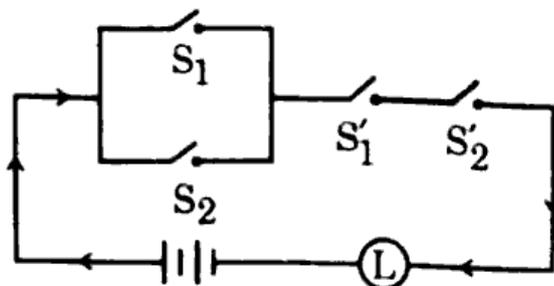
- (i) waiting time is between 1 to 3 minutes.
- (ii) waiting time is more than 4 minutes. (2)

SECTION – C

Attempt any EIGHT of the following questions :

[24]

- Q. 15. Express the following switching circuit in the symbolic form of logic. Construct the switching table and interpret it :



(3)

- Q. 16. Prove that : $2 \tan^{-1} \left(\frac{1}{3} \right) + \cos^{-1} \left(\frac{3}{5} \right) = \frac{\pi}{2}$. (3)
- Q. 17. In ΔABC if $a = 13$, $b = 14$, $c = 15$ then find the values of
(i) $\sec A$ (ii) $\operatorname{cosec} \frac{A}{2}$. (3)
- Q. 18. A line passes through the points $(6, -7, -1)$ and $(2, -3, 1)$. Find the direction ratios and the direction cosines of the line. Show that the line does not pass through the origin. (3)
- Q. 19. Find the cartesian and vector equations of the line passing through $A(1, 2, 3)$ and having direction ratios 2, 3, 7. (3)
- Q. 20. Find the vector equation of the plane passing through points $A(1, 1, 2)$, $B(0, 2, 3)$ and $C(4, 5, 6)$. (3)
- Q. 21. Find the n^{th} order derivative of $\log x$. (3)
- Q. 22. The displacement of a particle at time t is given by $s = 2t^3 - 5t^2 + 4t - 3$. Find the velocity and displacement at the time when the acceleration is 14 ft/sec^2 . (3)

- Q. 23. Find the equations of tangent and normal to the curve $y = 2x^3 - x^2 + 2$ at point $\left(\frac{1}{2}, 2\right)$. (3)
- Q. 24. Three coins are tossed simultaneously, X is the number of heads. Find the expected value and variance of X . (3)
- Q. 25. Solve the differential equation : $x \frac{dy}{dx} = x \cdot \tan\left(\frac{y}{x}\right) + y$. (3)
- Q. 26. Five cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability that : (3)
- (i) all the five cards are spades.
- (ii) none is spade.

SECTION - D

Attempt any FIVE of the following questions : [20]

- Q. 27. Find the inverse of $\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ by elementary row transformations. <https://www.maharashtrastudy.com> (4)
- Q. 28. Prove that homogeneous equation of degree two in x and y , $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - ab \geq 0$. Hence show that equation $x^2 + y^2 = 0$ does not represent a pair of lines. (4)
- Q. 29. Let \bar{a} and \bar{b} be non-collinear vectors. If vector \bar{r} is coplanar with \bar{a} and \bar{b} then show that there exist unique scalars t_1 and t_2 such that $\bar{r} = t_1\bar{a} + t_2\bar{b}$. For $\bar{r} = 2\hat{i} + 7\hat{j} + 9\hat{k}$, $\bar{a} = \hat{i} + 2\hat{j}$, $\bar{b} = \hat{j} + 3\hat{k}$, find t_1, t_2 . (4)

Q. 30. Solve the linear programming problem graphically.

Maximize : $z = 3x + 5y$

Subject to : $x + 4y \leq 24,$

$$3x + y \leq 21,$$

$$x + y \leq 9,$$

$$x \geq 0, y \geq 0$$

Also find the maximum value of z

(4)

Q. 31. If $x = f(t)$ and $y = g(t)$ are differentiable functions of t so that y is a function of x and if $\frac{dx}{dt} \neq 0$

then prove that $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$.

Hence find the derivative of 7^x w.r.t. x .

(4)

Q. 32. Evaluate : $\int e^{\sin^{-1} x} \left(\frac{x + \sqrt{1-x^2}}{\sqrt{1-x^2}} \right) dx$

(4)

Q. 33. Prove that : $\int_a^b f(x) dx = \int_a^b f(a+b-x) dx$

Hence evaluate : $\int_0^3 \frac{\sqrt{x}}{\sqrt{x} + \sqrt{3-x}} dx$

(4)

Q. 34. If a body cools from 80°C to 50°C at room temperature of 25°C in 30 minutes, find the temperature of the body after 1 hour.

(4)

