DAY – 09 SEAT NUMBER M 1 0 7 5 2 3

2026 | II | 21 | 1100 | **J-165** | (E)

MATHEMATICS & STATISTICS (40)
(ARTS & SCIENCE)

Time : 3 Hrs. (8 Pages) Max. Marks : 80

General instructions :

*The question paper is divided into **FOUR** sections.*

(1) ***Section A :*** *Q. 1 contains **Eight** multiple choice type questions, each carrying **Two** marks.*

*Q. 2 contains **Four** very short answer type questions, each carrying **One** mark.*

(2) ***Section B :*** *Q. 3 to Q. 14 contain **Twelve** short answer type questions, each carrying **Two** marks.*
*(Attempt any **Eight**)*

(3) ***Section C :*** *Q. 15 to Q. 26 contain **Twelve** short answer type questions, each carrying **Three** marks.*
*(Attempt any **Eight**)*

(4) ***Section D :*** *Q. 27 to Q. 34 contain **Eight** long answer type questions, each carrying **Four** marks.*
*(Attempt any **Five**)*

(5) *Use of log table is allowed. Use of calculator is not allowed.*

(6) *Figures to the right indicate full marks.*

(7) *Use of graph paper is not necessary. Only rough sketch of graph is expected.*

(8) *For each multiple choice type of question, only the first attempt will be considered for evaluation.*

(9) *Start answer to each section on a new page.*

SECTION – A

Q. 1. Select and write the correct answer of the following multiple choice type of questions : [16]

(i) The converse of contrapositive of $\sim p \rightarrow q$ is _____.

(a) $q \rightarrow p$ (b) $\sim q \rightarrow p$

(c) $p \rightarrow \sim q$ (d) $\sim q \rightarrow \sim p$ (2)

(ii) If $A = \begin{bmatrix} 2 & -4 \\ 3 & 1 \end{bmatrix}$, then the adjoint of matrix A is _____.

(a) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 3 \\ -4 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 4 \\ -3 & 2 \end{bmatrix}$ (2)

(iii) If $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$, then $x =$ _____.

(a) -1 (b) $\frac{1}{6}$

(c) $\frac{1}{3}$ (d) $\frac{3}{2}$ (2)

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(iv) The angle between the line $\bar{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}+\hat{j}+\hat{k})$ and the plane $\bar{r}\cdot(2\hat{i}-\hat{j}+\hat{k})=8$ is _____.

(a) $\sin^{-1}\left(\frac{\sqrt{2}}{3}\right)$ (b) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

(c) $\sin^{-1}\left(\frac{1}{2}\right)$ (d) $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2)

(v) If $y=\sec(\tan^{-1}x)$, then $\frac{dy}{dx}$ at $x=1$ is _____.

(a) $\frac{1}{2}$ (b) 1

(c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$ (2)

(vi) The approximate value of the function $f(x)=x^3-3x+5$ at $x=1.99$ is _____.

(a) 6.09 (b) 6.91

(c) 7.09 (d) 7.91 (2)

(vii) $\int_1^2 \frac{1}{x^2}\cdot e^{\frac{1}{x}}dx=$ _____.

(a) $\sqrt{e}+1$ (b) $\sqrt{e}-1$

(c) $\sqrt{e}(\sqrt{e}-1)$ (d) $\frac{\sqrt{e}-1}{e}$ (2)

$\int_1^2 \frac{1}{t^2}e^t dt$

$\frac{1}{x}=t$

$\frac{1}{x}$

$x \to t$

$1 \quad 1$

$2 \quad 1/2$

$\int_{1/2}^{1} e^t dt$

$=[e^t]_{1/2}^{1}$

$=e^1-e^{1/2}$

$=e-\sqrt{e}$

$=\sqrt{e}(\sqrt{e}-1)$

$t=\frac{1}{x}$

$\frac{dt}{dx}=\frac{-1}{x^2}$

$-dt=\frac{dx}{x^2}$

$\frac{1}{x}=t$

$-\frac{1}{x^2}=dt$

$\frac{1}{x^2}dx=-dt$

$\int_{1/2}^{1} e^t dt$

$=[e^t]_{1/2}^{1}$

$e^1-e^2=e^1+e^{2^2}$

(viii) If the p.d.f. of a continuous r.v. X is

$f(x) = \frac{x+2}{18}$, for $-2 < x < 4$

$= 0$, otherwise

then $P(|X| < 1) =$ _____

(a) $\frac{1}{9}$ (b) $\frac{2}{9}$

(c) $\frac{1}{27}$ (d) $\frac{2}{27}$ (2)

Q. 2. Answer the following questions : **[4]**

(i) Write the dual of $(p \vee q) \vee r \equiv p \vee (q \vee r)$ (1)

(ii) Evaluate : $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ (1)

(iii) Evaluate : $\int \frac{2x}{1+x^2} dx$ (1)

(iv) Write the degree of the differential equation

$(y''')^2 + 3(y'')^3 + 3xy' + 5y = 0$ (1)

SECTION – B

Attempt any EIGHT of the following questions : **[16]**

Q. 3. Construct the switching circuit of the statement pattern $(\sim p \wedge q) \vee (p \wedge \sim r)$. (2)

Q. 4. In ΔABC, prove that $a(b\cos C - c\cos B) = b^2 - c^2$. (2)

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Q. 5. Find the general solution of $4\cos^2\theta = 3$. (2)

Q. 6. Find k, if the sum of the slopes of the lines represented by $x^2 + kxy - 3y^2 = 0$ is twice their product. (2)

Q. 7. Find the value of p, for which the vectors $\bar{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\bar{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are perpendicular to each other. (2)

Q. 8. Find the vector equation of the line passing through the points $A(1,2,3)$ and $B(2,3,4)$. (2)

Q. 9. Find $\frac{dy}{dx}$, if $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (2)

Q. 10. Find $\frac{d^2y}{dx^2}$, if $y = x^3 + 7x^2 - 2x - 9$. (2)

Q. 11. Test whether the function $f(x) = x^3 + 6x^2 + 12x - 7$ is increasing or decreasing for all $x \in R$. (2)

Q. 12. A stone is dropped into a quiet lake and waves in the form of circles are generated. Radius of the circular wave increases at the rate of 3 cm/sec. How fast the area enclosed is increasing when the radius is 8 cm? (2)

Q. 13. Evaluate : $\int \sqrt{1+\sin 2x} \cdot dx$ (2)

Q. 14. Given that, $X \sim B(n,p)$, if $n = 10, E(X) = 8$ then find Var(X). (2)

SECTION – C

Attempt any EIGHT of the following questions : [24]

Q. 15. Examine whether the statement pattern $(p \wedge q) \wedge (\sim p \vee \sim q)$ is a tautology or contradiction or contingency. (3)

Q. 16. In ΔABC, if $A = 45°$, $B = 60°$ then find the ratio of its sides. (3)

Q. 17. If two vertices of a triangle are $A(3,1,4)$ and $B(-4,5,-3)$ and the centroid of the triangle is $G(-1,2,1)$, then find the coordinates of the third vertex C of the triangle. (3)

Q. 18. If D, E, F are the mid-points of the sides BC, CA, AB respectively of ΔABC, then prove that $\overline{AD} + \overline{BE} + \overline{CF} = \bar{0}$. (3)

Q. 19. Show that the lines $\bar{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ and $\bar{r} = (4\hat{i} - 3\hat{j} + 2\hat{k}) + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$ intersect each other. (3)

Q. 20. Find the cartesian equation of the plane $\bar{r} = (\hat{i} - \hat{j}) + \lambda(\hat{i} + \hat{j} + \hat{k}) + \mu(\hat{i} - 2\hat{j} + 3\hat{k})$. (3)

Q. 21. If $y = f(u)$ is a differentiable function of u and $u = g(x)$ is a differentiable function of x then prove that y is a differentiable function of x and $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$. (3)

Q. 22. Verify LMVT for the function $f(x) = \log x$, on $[1, e]$. (3)

Q. 23. Evaluate : $\int \frac{\sin x}{\sin 3x} dx$ (3)

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Q. 24. Solve the D.E. $3e^x \tan y\,dx+(1+e^x)\sec^2 y\,dy=0$. (3)

Q. 25. Find $E(X)$ and $V(X)$, where X is the number obtained on uppermost face, when a fair die is thrown. (3)

Q. 26. A pair of dice is thrown 4 times. If getting a doublet is considered as success, find the probability of two successes. (3)

SECTION – D

Attempt any FIVE of the following questions : **[20]**

Q. 27. If $A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}$, prove that $A\cdot(\text{adj } A)=(\text{adj } A)\cdot A=|A|\cdot I$ (4)

Q. 28. Show that every homogeneous equation of degree two in x and y i.e. $ax^2+2hxy+by^2=0$, represents a pair of lines passing through the origin, if $h^2-ab\geq 0$ (4)

Q. 29. If $A(\bar{a})$ and $B(\bar{b})$ are any two points in space and $R(\bar{r})$ be a point on the line segment AB dividing internally in the ratio $m:n$ then prove that $\bar{r}=\dfrac{m\bar{b}+n\bar{a}}{m+n}$. (4)

Q. 30. Solve the L.P.P. graphically :

Minimize : $z=5x+2y$,

Subject to , $5x+y\geq 10,\ x+y\geq 6,$

$x\geq 0,\ y\geq 0$ (4)

Q. 31. Evaluate : $\int \frac{3x^2+4x-5}{(x^2-1)(x+2)} \cdot dx$ (4)

Q. 32. Prove that : $\int_a^b f(x)\,dx = \int_a^b f(a+b-x) \cdot dx$

Hence, find $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin^2 x \cdot dx$ (4)

Q. 33. Find the area enclosed between the circle $x^2+y^2=1$ and the line $x+y=1$ lying in the first quadrant. (4)

Q. 34. Solve the differential equation $x^2 \cdot \frac{dy}{dx} = x^2+xy+y^2$. (4)

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